

## Multi-Solid Varieties and Mh-transducers

Slavcho Shtrakov

Communicated by B.V. Novikov

**ABSTRACT.** We consider the concepts of colored terms and multi-hypersubstitutions. If  $t \in W_\tau(X)$  is a term of type  $\tau$ , then any mapping  $\alpha_t : Pos^{\mathcal{F}}(t) \rightarrow \mathbb{N}$  of the non-variable positions of a term into the set of natural numbers is called a coloration of  $t$ . The set  $W_\tau^c(X)$  of colored terms consists of all pairs  $\langle t, \alpha_t \rangle$ . Hypersubstitutions are maps which assign to each operation symbol a term with the same arity. If  $M$  is a monoid of hypersubstitutions then any sequence  $\rho = (\sigma_1, \sigma_2, \dots)$  is a mapping  $\rho : \mathbb{N} \rightarrow M$ , called a multi-hypersubstitution over  $M$ . An identity  $t \approx s$ , satisfied in a variety  $V$  is an  $M$ -multi-hyperidentity if its images  $\rho[t \approx s]$  are also satisfied in  $V$  for all  $\rho \in M$ . A variety  $V$  is  $M$ -multi-solid, if all its identities are  $M$ -multi-hyperidentities. We prove a series of inclusions and equations concerning  $M$ -multi-solid varieties. Finally we give an automata realization of multi-hypersubstitutions and colored terms.

### References

- [1] G. Birkhoff, *Lattice theory*, (3rd ed.) Amer. Math. Soc., Providence, 1967
- [2] S. Burris and H. Sankappanavar, *A Course in Universal Algebra*, The millennium edition, 2000
- [3] H. Comon, M. Dauchet, R. Gilleron, F. Jacquemard, D. Lugiez, S. Tison, M. Tommasi, *Tree Automata, Techniques and Applications*, 1999, <http://www.grappa.univ-lille3.fr/tata/>
- [4] K. Denecke, J. Koppitz and Sl. Shtrakov, Multi-Hypersubstitutions and Coloured Solid Varieties, *J. Algebra and Computation*, J. Algebra and Computation, Volume 16, Number 4, August, 2006, pp.797-815.

---

**2001 Mathematics Subject Classification:** 08B15, 03C05, 08A70.

**Key words and phrases:** Colored term; multi-hypersubstitution; deduction of identities.

- [5] K. Denecke, D. Lau, R. Pöschel and D. Schweigert, Hyperidentities, Hyperequational Classes and Clone Congruences, *General Algebra 7*, Verlag Hölder-Pichler-Tempsky, Wien 1991, Verlag B.G. Teubner Stuttgart, pp.97-118
- [6] F. Gécseg, M. Steinby, *Tree Automata*, Akadémiai Kiadó, Budapest 1984
- [7] E. Graczyńska, *On connection between identities and hyperidentities*, Bull.Sect.Logic 17(1988),34-41.
- [8] G. Gratzer, *Universal Algebra*, D. van Nostrand Co., Princetown, 1968.
- [9] R. McKenzie, G. Mc Nulty and W. Taylor, *Algebras, Lattices, Varieties*, Vol. I, Belmont, California 1987.
- [10] J. W. Thatcher and J.B. Wright, Generalized finite automata, *Notices Amer. Math. Soc.*, **12**. (1965), abstract No. 65T-649,820.

#### CONTACT INFORMATION

Dept. of Computer Sciences, South-West  
University, 2700 Blagoevgrad, Bulgaria  
*E-Mail:* [shtrakov@aix.swu.bg](mailto:shtrakov@aix.swu.bg)  
*URL:* <http://home.swu.bg/shtrakov>