On the complexity of finite valued functions

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Understanding the complexity of k-valued functions is still one of the fundamental tasks in the theory of computation. The essential variables in a function f are defined as variables which occur in f and weigh with the values of that function. The number of essential variables is a measure of complexity for discrete functions and any opportunity to reduce the complexity is an important procedure in theoretical and applied computer science and modeling. When replacing some variables in a function with constants the resulting functions are called *subfunctions*, and when replacing all essential variables in a function with constants we obtain an *implementation* of this function. Such an implementation corresponds with a path in an *ordered decision diagram* (ODD) (see [1]) of the function which connects the root with a leaf of the diagram. The sets of essential variables in subfunctions of a given function are called *separable sets*.

By $Sub(f)$, $Imp(f)$ and $Sep(f)$ we denote the sets of subfunctions, implementations and separable sets, respectively in the function f. The cardinalities of these sets are complexity measures for the corresponding functions. We introduce three equivalence relations $f \simeq_{imp} g$, $f \simeq_{sub} g$ and $f \simeq_{sep} g$ in P_k^n defined by the equations $|Imp(f)| = |Imp(g)|$, $|Sub(f)| = |Sub(g)|$ and $|Sep(f)| = |Sep(g)|$.

Theorem $(i)\ \simeq_{imp} \leq \simeq_{sep}$; $(ii)\ \simeq_{sub} \leq \simeq_{sep}$;
$(iii)\ \simeq_{imp} \not\leq \simeq_{sub}$; $(iv)\ \simeq_{sub} \not\leq \simeq_{imp}$.

Let us denote by Im_k^n, Sb_k^n and Sp_k^n the transformation groups induced by the equivalence relations $\simeq_{imp}$, $\simeq_{sub}$ and $\simeq_{sep}$, respectively. We compare groups Im_2^n, Sb_2^n and Sp_2^n with the lattice of the restricted affine groups (RAG) studied in [2]. The number of equivalence classes $t(G)$ in cases of n-ary boolean functions for $n = 3$ and $n = 4$ are calculated. We also obtain upper bounds in cases of $n \leq 6$.

n	$t(G_2^n)$	$t(Im_2^n)$	$t(Sb_2^n)$	$t(Sp_2^n)$
1	3	2	2	2
2	6	4	4	3
3	22	13	11	5
4	402	104	74	11
5	1 228 158	< 1 228 158		
6	400 507 806 843 728	< 400 507 806 843 728		

Number of classes under *symmetry type*, $\simeq_{imp}$, $\simeq_{sub}$ and $\simeq_{sep}$.

References

[1] Bryant, R. E., Graph-based algorithms for Boolean function manipulation. *IEEE Transactions on Computers*, C-35(8), (1986), pp. 677–691.

[2] Lechner, R. J., Harmonic analysis of switching functions. *Recent Developments in Switching Theory*, (ed. A. Mikhopadhyay), NY, Academic Press, (1971), pp. 121-228.